

Discussion of “Effects of Errors in Coordinates on Transformation Parameters” by Hakan S. Kutoglu, Cetin Mekik, and Hakan Akcin

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We read the article with great interest. One of our main objections to the piece is that the authors did not sufficiently incorporate or reference previous work published on the topic. The other problem, perhaps of greater significance, is that the authors do not correct inconsistencies encountered in some of the references cited. Most of the matrix expressions explicitly written in the article were originally given in Soler (1976). Our later work (Soler and van Gelder 1987) presents the same type of equations used in the paper to compose Figs. 1, 2, and 3, but also incorporates important additions that the authors of the new paper neglected. Still further, a simplified version of these same equations appears in the appendix of a tutorial article published in the *Journal of Surveying Engineering* (Soler and Hothem 1988).

The advantage of using the formulation in Soler and van Gelder (1987) is fourfold:

- The mathematical expression introduced is given in a more general way as a function of the orthogonal matrix \mathbf{R} that rotates the local geocentric frame into the local geodetic frame. This approach reduces the total number of matrices to be written explicitly.
- The components of the differential changes in geodetic longitude, latitude, and ellipsoid height are given in linear units as opposed to angular units, a more practical alternative when differential quantities are involved.
- The corrections caused on the curvilinear coordinates by differential changes of the semimajor axes δa and flattening δf were expanded to second-order terms.
- Finally, and most important, the recently published article neglects scaling the differential changes of the semimajor axis δa when the three-dimensional scale of the Euclidean space of our ordinary experience is changed by δk .

Paraphrasing the text from Soler and van Gelder (1987), two different types of scale change may be considered: A global or scale change represented by δk that primarily affects the unit of length along the three Cartesian axes and an apparent geodetic

network scale change influenced by δa . Both changes are possible and compatible. If we know δa and δk , they can and should be included simultaneously in deterministic equations of the type discussed above by redefining the total change in semimajor axis by

$$\delta a_r = \delta a + a\delta k \quad (1)$$

This very point has not been addressed often, although it may occasionally be of critical importance. It should be emphasized here that a change of δk will not physically modify the size of the reference ellipsoid, although the actual magnitude of the semimajor axis will differ because the new basic “measuring yardstick” has a different unit of length.

On the contrary, a change δa will leave the scale of three-dimensional space intact, but the physical size of the ellipsoid defining the datum in question will be modified. Consequently, any geodetic quantity related to points on the ellipsoid (ellipsoidal cord distances, geodesics, normal sections, geodetic heights, undulations, etc.) will change in magnitude, although the unit of length in which they were measured—that is, the scale—remains the same before and after the change of semimajor axis is implemented. Nevertheless, spatial distances between points not on the ellipsoid or physical parameters independent of the reference ellipsoid such as orthometric heights H will remain invariant. Thus, it may be concluded that a δa change is equivalent to an apparent datum or network scale change although the scale of the space remains constant.

In conclusion, Soler (1976) illuminates the mathematical fact that the Jacobian \mathbf{J} between curvilinear and Cartesian coordinates can be obtained analytically in a simple manner, even where simple expressions for the geodetic (ellipsoidal) coordinates as a function of the Cartesian coordinates are absent: The Jacobian \mathbf{J} is the product of the rotation matrix \mathbf{R} and a metric matrix \mathbf{H} . To avoid the introduction of scaling errors into the curvilinear coordinates when differential changes δa and δk are involved, all transformations should be done in Cartesian coordinates and the conversion between Cartesian and curvilinear coordinates should be implemented only at the very end using the new adopted parameters of the rotational ellipsoid, if necessary.

References

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